NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3772

EFFECT OF THREE DESIGN PARAMETERS ON THE OPERATING

CHARACTERISTICS OF 75-MILLIMETER-BORE CYLINDRICAL

ROLLER BEARINGS AT HIGH SPEEDS

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ISTICS OF 75-MILLIMETER-BORE CYLINDRICAL ROLLER

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SUMMARY

Three experimental investigations were conducted with 75-millimeter-bore (size 215) cylindrical roller bearings to determine the effects of roller axial clearance ratio, cage diametral clearance ratio, and cage land (locating surface) width-diameter ratio on bearing operating temperatures, wear, cage slip, and high-speed operating characteristics. Thirty test bearings were operated over a range of DN (product of bearing bore in mm and shaft speed in rpm) values from 0.3×10^6 to 2.1×10^6, at radial loads of 7, 113, and 368 pounds, and at oil flows of 2.0, 2.75, and 5.5 pounds per minute.

Two bearings with a roller axial clearance ratio (ratio of roller axial clearance to roller length) of 0.0005 failed because of seizure of the rollers in the inner-race track. Roller axial clearance ratios in the range 0.0006 to 0.0061 had no significant effect on bearing operating temperature, wear, cage slip, or high-speed operating characteristics.

Bearing temperatures were minimum at a cage diametral clearance ratio (ratio of cage diametral clearance to cage locating-surface diameter) of about 0.004. Inner-race temperature was more sensitive to changes in this ratio than was outer-race temperature because variations in heat generation caused by changes in cage design occurred at the inner race. An increase in cage diametral clearance ratio increased cage wear and also increased cage slip at a load of 7 pounds and a DN value of 1.2×10^6. Bearings with large ratios showed poorer high-speed operating characteristics than bearings with small ratios.

Bearing inner-race temperatures decreased slightly with increasing cage land width-diameter ratio (ratio of cage locating-surface width to cage locating-surface diameter), but outer-race temperatures were

unaffected. Increasing this ratio increased cage wear, indicating that boundary lubrication conditions prevailed at the cage locating surfaces of all the cages tested, and also decreased cage slip, possibly because of increased drag between the cage and inner race. Bearings with large ratios showed poorer high-speed operating characteristics.

INTRODUCTION

The geometry of a rolling contact bearing is extremely important in determining its operating characteristics. Therefore, it is important, in bearing applications as severe as those in aircraft turbine engines, to design bearings with the optimum geometry to minimize heat generation and wear and thus to contribute toward maximum bearing life. Unfortunately, the design of some engines, rather than the operating conditions, dictates the type of bearing that must be used to facilitate assembly. There are many geometric variables in a cylindrical roller bearing and a vast experimental program would be required to determine the effect of each of these on bearing performance. Some of the variables that may appreciably affect performance are the number of rollers, the roller length-diameter ratio, the roller axial clearance in the roller track, the bearing diametral clearance, the cage diametral clearance, the cage locating-surface width, and the roller clearances in the cage pocket.

The fact that cage failures have ranked high among the causes of roller-bearing failures (refs. 1 to 6) may be an indication that improvements in bearing and cage geometry are needed. The cage problem is certainly partly one of materials, but improvements in performance can be achieved by altering cage geometry (ref. 7). Similar improvements in performance might be achieved through further changes in the geometry of the bearing components. Work on the effects of bearing geometry on the performance of rolling contact bearings is reported in references 8 to 12.

The experimental results reported herein were obtained from investigations conducted at the NACA Lewis laboratory. The effects of roller axial clearance ratio, cage diametral clearance ratio, and cage (locating surface) land width-diameter ratio on bearing performance were determined. Operating temperature, wear, and cage slip were used as performance criteria for 75-millimeter-bore cylindrical roller bearings. A total of 30 bearings was operated at DN values of 0.3×10^6 to 2.1×10^6, radial loads of 7 to 368 pounds, and oil flows of 2.0 to 5.5 pounds per minute. Detailed descriptions of the test bearings are given in the section TEST BEARINGS.

APPARATUS

Bearing Rig

The bearing rig (fig. 1) used for this investigation is the same as that used in references 8 and 13, modified according to reference 7. The bearing was mounted on one end of the test shaft, which was supported in a cantilever fashion in order that bearing component parts and lubricant flow could be observed during operation. A radial load was so applied to the test bearing by means of a lever and dead-weight system that the aline-ment of the outer race of the test bearing was essentially unaffected by small shaft deflections or by small shaft and load-arm misalinements.

The support bearings were lubricated in the manner described in ref-erence 8. The oil was supplied to the support bearings at a pressure of 10 pounds per square inch through a 0.180-inch-diameter jet and at a tem-perature of 100° F.

Drive Equipment

The high-speed drive equipment consisted of a shunt-wound 30-horsepower direct-current motor connected to a 14:1 speed increaser. The high-speed shaft of the speed increaser was connected to the test shaft by means of a floating spline coupling. The speed range of the test shaft was 1100 to 36,000 rpm, controllable to within ±50 rpm at all speeds.

Temperature Measurement

Iron-constantan thermocouples were located in the outer-race housing at 60° intervals around the outer-race periphery at the axial midpoint of the bearing under investigation. A copper-constantan thermocouple was pressed against the bore of the inner race at the axial midpoint of the test bearing. The thermocouple electromotive force was transmitted from the rotating shaft by means of small slip rings located on the end of the test shaft (ref. 14).

Lubrication System

The lubrication system used was of the circulating type. Separate pumps were used to supply oil to the support bearings and the test bear-ing, and full-flow filters were provided after the oil-supply pumps. Oil inlet temperature was controlled to within ±1° F and oil inlet pressure to within ±0.5 pound per square inch. Support-bearing oil was drained by gravity from the base of the rig. Test-bearing oil was collected in cans and pumped back to the sump.

TEST BEARINGS

All test bearings used in these investigations were size 215 cylindrical roller bearings with 75-millimeter bores, 130-millimeter outside diameters, and 25-millimeter widths.

Roller Axial Clearance Bearings

A total of 14 bearings, each equipped with a one-piece inner-race-riding bronze cage, was used in this investigation. Each bearing had 18 rollers of 0.6537-inch nominal length and 0.5258-inch nominal diameter. Roller axial clearance (difference between inner-race roller track width and roller length) was varied from 0.0003 to 0.0040 inch. These roller axial clearances correspond to roller axial clearance ratios (ratio of roller axial clearance to roller length) of 0.0005 to 0.0061, respectively. Roller length was varied to obtain the range of roller axial clearance ratio used. The standard roller axial clearance ratio for these bearings is about 0.0038. An effort was made to keep all other important dimensional variables constant among this group of bearings. A summary of the important dimensions of these bearings is given in table I(a), and a schematic diagram is given in figure 2(a). The races of these bearings were made of SAE 4620 steel, while the rollers were made of SAE 52100 steel. Both races and rollers were hardened to a minimum hardness of Rockwell C-58.

Cage Diametral Clearance Bearings

Ten bearings, each equipped with a one-piece inner-race-riding leaded brass cage, were used in this investigation. Each bearing had 18 rollers of 0.5510-inch nominal length and 0.5510-inch nominal diameter. These bearings had cage diametral clearances (difference between cage inside diameter and cage locating-surface diameter on inner race) of 0.004 (two bearings), 0.008, 0.009, 0.012, 0.013 (two bearings), 0.016, 0.021, and 0.022 inch. These cage diametral clearances correspond to cage diametral clearance ratios (ratio of cage diametral clearance to cage inside diameter) of 0.0011 to 0.0059. The cage inside diameter was varied to give the desired clearance ratio. The standard cage diametral clearance ratio for these bearings is about 0.0038. These bearings were selectively assembled to keep all other important dimensional variables constant. A summary of the important dimensions of these bearings is given in table I(b) and a schematic diagram is given in figure 2(b). Races and rollers were made of SAE 52100 steel hardened to a minimum hardness of Rockwell C-58.

Cage Land Width Bearings

Eight bearings (including two from the cage diametral clearance investigation), each equipped with a one-piece inner-race-riding leaded brass cage, were used in this investigation. Each bearing had 18 rollers of 0.5510-inch nominal length and 0.5510-inch nominal diameter. These bearings had cage land (locating surface) widths of 0.130, 0.151, 0.171, and 0.192 inch (two each). These cage land widths correspond to cage width-diameter ratios (ratio of cage land width to cage inside diameter) of 0.035 to 0.052. The standard cage land width-diameter ratio for these bearings is about 0.035. As with the cage diametral clearance bearings, selective assembly was used to keep all other important dimensional variables constant. The weights of the cages in this series of bearings were kept constant by maintaining a constant cage cross-sectional area as shown in figure 2(c). A summary of the important dimensions of these bearings is given in table I(c). Races and rollers were made of SAE 52100 steel hardened to a minimum hardness of Rockwell C-58.

PROCEDURE

Order of Test

Roller axial clearance bearings. - The bearings in this group were first subjected to a series of tests at oil flows of 2.0, 2.75, and 5.5 pounds per minute and a radial load of 368 pounds at DN values up to 1.2×10^6. The running time at each DN value is given in table II(a). Each bearing was then removed from the test rig for visual examination and inspection for wear as determined by weight loss. They were reinstalled in the test rig and run at DN values from 1.2×10^6 to 2.1×10^6 at a load of 368 pounds and oil flows of 2.0, 2.75, and 5.5 pounds per minute (table II(b)). Then, they were again removed from the test rig for visual examination and inspection for wear.

Finally, selected bearings in the group were reinstalled in the test rig and subjected to a series of cage-slip tests. These tests were run at DN values of 1.2×10^6 and 1.5×10^6, an oil flow of 2.75 pounds per minute, and radial loads of 7, 113, and 368 pounds.

Cage diametral clearance and cage land width bearings. - The bearings in both of these groups were run at oil flows of 2.0 and 2.75 pounds per minute, at a radial load of 368 pounds, and at DN values of 0.3×10^6, 0.735×10^6, 0.995×10^6, and 1.2×10^6. The bearings were run at each oil flow and DN value for exactly 0.5 hour, after which they were run at an oil flow of 2.75 pounds per minute, a load of 368 pounds, and a DN value of 1.2×10^6 until the total running time had reached 20 hours. The bearings were then removed from the test rig for visual examination and inspection for wear as determined by weight loss.

Selected bearings in each group were then reinstalled and subjected to the same series of cage-slip tests as were the roller axial clearance bearings.

Lubrication of Test Bearings

Lubricant was supplied to the test bearings through a single jet having a 0.050-inch-diameter orifice. The oil was directed normal to the bearing face at the cage locating surface. When the roller axial clearance bearings were tested, oil was scavenged from both sides of the test bearing. However, when the cage diametral clearance and cage land width bearings were tested, the oil collector on the jet side of the test bearing was plugged prior to tests. This forced all the oil to flow through the test bearing to be scavenged from the side opposite the jet. This system of lubrication, sometimes called "puddling," was necessary to eliminate the effects of varying cage diametral clearance and cage land width on the fraction of oil flow that flows through the test bearing.

A highly refined nonpolymer-containing petroleum-base lubricating oil used to lubricate the bearings in several current turbojet engines was used to lubricate the test bearings. The test oil (MIL-O-6081A, grade 1010) has a viscosity of 10 centistokes at 100° F and 2.6 centistokes at 210° F. The oil used came from several drums, but regular checks of neutralization number revealed no significant differences.

Oil was supplied to the test bearings at a temperature of 100° F and at pressures of 30 to 210 pounds per square inch, which corresponded to oil flows of 2.0 to 5.5 pounds per minute, respectively.

Test-Bearing Measurements

Measurements of test-bearing component parts were obtained in a constant-temperature gage room. Standard precision inspection instruments were used to obtain the dimensions of the test bearings in table I.

RESULTS AND DISCUSSION

Roller Axial Clearance Investigation

Bearing failures. - Of the 14 bearings tested in this investigation, 12 operated satisfactorily to a DN value of 1.8×10^6. Bearings 43 and 44 (see tables I(a) and II) failed at DN values of 1.2×10^6 (16,000 rpm) and 1.65×10^6 (22,000 rpm), respectively. Post test examination of these bearings indicated that both failures were caused by seizure of the rollers

in the inner-race roller track because of differential expansion of the rollers and inner race. Both bearings 43 and 44 had a roller axial clearance ratio of 0.0005. Calculations indicated that, at the time of failure, roller temperatures were at least 65° F higher than the inner-race temperature. The fact that bearing 45, with a roller axial clearance ratio of 0.0006, ran satisfactorily indicates that even this small ratio is sufficient under the conditions of these tests.

Bearing operating temperatures. - Figure 3 shows that, within the range of roller axial clearance ratios investigated, the effect of roller axial clearance ratio on both bearing outer-race-maximum and inner-race temperatures is negligible. Curves are shown for DN values of 0.735×10^6, 0.995×10^6, 1.2×10^6, 1.5×10^6, and 1.8×10^6 at oil flows of 2.0, 2.75, and 5.5 pounds per minute. Inner-race temperatures are not measured at DN values over 1.2×10^6 because the excessive sliding velocities at the slip rings cause erratic readings. In all tests the load was 368 pounds and the oil inlet temperature was 100° F. Inner-race temperatures show some tendency to decrease with increasing roller axial clearance ratio, but the decrease is not significant in view of the scatter in test data.

Bearing wear. - The wear data (obtained by determining weight loss) for the roller axial clearance bearings are shown in table III(a). As with most wear data, there is considerable scatter. The data indicate that roller axial clearance ratio does not influence bearing wear.

Bearing cage slip. - Cage-slip data for six of the roller axial clearance bearings are shown in table III(b). Cage slip is defined as follows:

$$\text{Percentage cage slip} = \left(\frac{N_{c,t} - N_c}{N_{c,t}} \right) 100$$

(All symbols are defined in appendix A.) These data were obtained at DN values of 1.2×10^6 and 1.5×10^6 and loads of 7, 113, and 368 pounds. All these bearings show fairly high values of cage slip at the two lighter loads, but roller axial clearance ratio does not seem to influence cage slip in the range of ratios from 0.0021 to 0.0061.

Function of roller axial clearance ratio. - The roller axial clearance ratio in a cylindrical roller bearing is one of the factors that determine the maximum skew angle of the rollers. The relation between this ratio and the maximum roller skew angle in terms of pertinent geometric variables is derived in appendix B. A plot of maximum roller skew angle as a function of roller axial clearance ratio for the bearings in this investigation (based on appendix B) is shown in figure 4. Within the range of roller axial clearance ratios from 0 to 0.008 the relation

is linear. The maximum roller skew angle possible in the bearings investigated was about 0.64°. Apparently this degree of roller skewing does not significantly affect the bearing operating characteristics.

Thus, bearing performance (in terms of bearing temperature, wear, and cage slip) is unaffected by roller axial clearance ratios in the range 0.0006 to 0.0061. This substantiates the findings of reference 12, that roller axial clearances from 0.001 to 0.006 inch had negligible effect on bearing friction and heat generation in 2-inch-bore roller bearings. It may be that these test conditions did not include the conditions under which the roller axial clearance ratio becomes an important variable. These conditions might involve very light loads under which the rollers could skew freely, or misalinement that would force the rollers to skew.

Cage Diametral Clearance Investigation

<u>Bearing operating temperatures</u>. - The effect of cage diametral clearance ratio on bearing outer-race-maximum and inner-race temperatures is shown in figure 5. Curves are shown for DN values of 0.735×10^6, 0.995×10^6, and 1.2×10^6 at oil flows of 2.0 and 2.75 pounds per minute. In all tests the load was 368 pounds and the oil inlet temperature was 100° F.

At oil flows of 2.0 and 2.75 pounds per minute, both outer-race-maximum and inner-race temperatures are minimum at a cage diametral clearance ratio of about 0.004. As would be expected with bearings with inner-race riding cages, inner-race temperatures are much more sensitive to changes in the cage diametral clearance ratio than are outer-race temperatures.

<u>Bearing wear</u>. - The cage-wear data for the cage diametral clearance bearings are shown in table IV(a) and in figure 6. Cage wear shows a tendency to increase and to become more erratic with increasing cage diametral clearance ratio. There does not seem to be any relation between cage wear (fig. 6) and bearing operating temperature (fig. 5). An analysis of the load capacity of extremely short journal bearings that support the cage seems impossible because of the complex nature of the loads on the cage. However, it would appear that the cages with smaller cage diametral clearance ratios operated at conditions closer to hydrodynamic than did the cages with larger ratios. Kreisle (ref. 15) reports the performance of short journal bearings with length-diameter ratios as low as 0.0187. In reference 15 it is shown that, for constant load and speed, minimum film thickness is practically constant for $C_d/D > 0.001$ and $h_m/D < 0.0002$. The cage diametral clearance bearings for which performance is reported had values of C_d/D from 0.0011 to 0.0059. The analysis of

reference 15 assumes a constant bearing load over the range of diametral clearances. Such is not the case for a bearing cage since the cage load is affected by its eccentricity. The maximum cage eccentricity is, in turn, a function of the cage diametral clearance. If the effect of cage eccentricity on cage load is appreciable, the theoretical minimum film thickness would decrease with increasing cage diametral clearance ratio. The cage-wear data seems to indicate that this is the case.

Bearing cage slip. - Bearing cage-slip data for the cage diametral clearance bearings at loads of 7, 113, and 368 pounds and DN values of 1.2×10^6 and 1.5×10^6 are shown in table IV(b). Bearings 61 and 65 (cage diametral clearance ratio, 0.0011) show less tendency to slip than did the bearings with larger cage diametral clearance ratios; but, because of the scatter present in data of this type, the differences are probably not significant. Bearings 62 and 63 (cage diametral clearance ratios, 0.0024 and 0.0057, respectively) show wide variations in cage slip at a load of 7 pounds and a DN value of 1.2×10^6. Considerable variations in bearing temperature accompanied the variations in cage slip. All the bearings in this group showed less cage slip at a DN value of 1.5×10^6 than at 1.2×10^6.

Bearing high-speed operating characteristics. - Cage diametral clearance ratio has a significant effect on the high-speed operating characteristics of the bearing. Bearings 63 and 69 (cage diametral clearance ratios, 0.0057 and 0.0059, respectively) failed at a DN value of 1.5×10^6 (20,000 rpm), whereas bearings 68 and 64 (cage diametral clearance ratios, 0.0032 and 0.0035, respectively) ran satisfactorily at a DN value of 1.8×10^6 (24,000 rpm). The incipient failures of bearings 63 and 69 were accompanied by severe vibration. The large cage clearances of these bearings may have contributed to the vibration because they permit greater cage eccentricities, which probably create the same effect as cage unbalance.

Cage Land Width Investigation

Bearing operating temperatures. - The effect of cage land (locating surface) width-diameter ratio on bearing outer-race-maximum and inner-race temperatures is shown in figure 7. Curves are shown for DN values of 0.735×10^6, 0.995×10^6, and 1.2×10^6 at oil flows of 2.0 and 2.75 pounds per minute. The load was 368 pounds and the oil-inlet temperature was 100° F in all tests.

At both oil flows, inner-race temperatures decrease with increasing cage land width. No effect on bearing outer-race-maximum temperature is apparent. This may be explained by the fact that any change in bearing characteristics produced by changes in the cage locating-surface width

would affect the inner-race temperature much more than the outer-race temperature because the heat generation due to changes in geometry is taking place at the cage inner-race interface. These data show that there is a slight reduction in heat generation with increasing cage locating-surface width.

Bearing wear. - The cage wear data for the cage land width bearings are shown in table V(a) and figure 8. These data show that there is a tendency for cage wear to increase with increasing land width. Reference 15 shows that with load, speed, and clearance constant the minimum film thickness of a short journal bearing will increase with increasing width. Prior to this investigation it was hoped that hydrodynamic lubrication at the cage locating surface could be achieved by utilizing the maximum cage locating-surface width available. The fact that wear increased with cage locating-surface width indicates that this goal was not achieved. Boundary lubrication conditions evidently prevailed at the cage locating surface of all the cages tested.

Bearing cage slip. - Bearing cage-slip data for the cage land width bearings at loads of 7, 113, and 368 pounds at DN values of 1.2×10^6 and 1.5×10^6 are shown in table V(b). Bearings 62 and 67 (cage land width-diameter ratio, 0.035) showed higher cage slip at loads of 7 and 113 pounds and a DN value of 1.2×10^6 than did the bearings with higher ratios. Since the cage wear for these bearings increased with increasing cage land width-diameter ratio (table V(a) and fig. 8), indicating that boundary lubrication was present at the cage locating surface, the low values of cage slip for the bearings with the three highest ratios may have been due to increased drag between the cage and inner race. Bearing 75, in particular, showed negative cage slip at all three loads and at a DN value of 1.2×10^6. Negative cage slip in a bearing with an inner-race-riding cage can be caused by excessive drag between the cage and inner race.

Bearing high-speed operating characteristics. - Increasing the cage land width appears to have a detrimental effect on the high-speed operating characteristics of the bearing. Bearing 75 (cage land width-diameter ratio, 0.046) and bearings 71 and 77 (cage land width-diameter ratios, 0.052) all failed at a DN value of 1.5×10^6 (20,000 rpm). The remaining bearings in this group ran without failure at a DN value of 1.5×10^6 although bearing 67 (ratio, 0.035) ran rough at a DN value of 1.5×10^6 and a load of 113 pounds.

SUMMARY OF RESULTS

In three separate experimental investigations, a total of thirty 75-millimeter-bore (size 215) cylindrical roller bearings were used to determine the effects of roller axial (end) clearance, cage diametral clearance, and cage land (locating surface) width. Test bearings were operated over a range of DN values from 0.3×10^6 to 2.1×10^6, at loads of 7, 113, and 368 pounds, and at oil flows of 2.0, 2.75, and 5.5 pounds per minute with the following results:

1. Roller axial clearance ratios (ratio of roller axial clearance to roller length) in the range 0.0006 to 0.0061 had no significant effect on bearing operating temperature, wear, or cage slip. Two bearings with a roller axial clearance ratio of 0.0005 failed because of seizure of the rollers in the inner-race track.

2. Bearing temperatures were minimum at a cage diametral clearance ratio (ratio of cage diametral clearance to cage locating-surface diameter) of about 0.004. Inner-race temperature was more sensitive to changes in this ratio than was outer-race temperature because variations in heat generation caused by changes in cage design occurred at the inner race. Increasing this ratio increased cage wear and also increased cage slip at a load of 7 pounds and a DN value of 1.2×10^6. Large cage diametral clearance ratios were detrimental to good high-speed operating characteristics; for example, two bearings with ratios of 0.0057 and 0.0059 failed at a DN value of 1.5×10^6.

3. Cage land (locating surface) width-diameter ratio (ratio of cage-locating-surface width to cage-locating-surface diameter) had a negligible effect on bearing outer-race temperatures, while inner-race temperatures decreased slightly with increasing cage land width-diameter ratio. An increased cage land width-diameter ratio increased cage wear, which indicated that boundary lubrication conditions prevailed at the cage locating surface of all the cages tested, and also decreased bearing cage slip, probably because of increased drag between the cage and inner race. Bearings with large ratios showed poorer high-speed operating characteristics.

Lewis Flight Propulsion Laboratory
    National Advisory Committee for Aeronautics
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APPENDIX A

SYMBOLS

a radius of roller face at point where corner radius blends into face

b chord length of inner-race flange outside diameter contacted by roller face

C_d diametral clearance

D bearing diameter

d diameter

h_m minimum film thickness

l length or width

Δl roller axial clearance

N_c actual cage angular velocity

$N_{c,t}$ theoretical cage angular velocity

R radius of outside diameter of inner-race flange

r radius of roller track

S roller radius

α arc tan b/l_r

β roller skew angle

γ $\alpha - \beta$

ζ roller axial clearance ratio $(\Delta l/l_r)$

θ arc sin $\dfrac{b}{2a}$

Subscripts:

c   cage

r   roller

s   cage locating surface

t   inner-race track

APPENDIX B

DERIVATION OF MAXIMUM ROLLER SKEW ANGLE AS A FUNCTION

OF ROLLER AXIAL CLEARANCE RATIO

The exact geometry involved in calculating the roller skew angle is complex; but, for the realistic case of small β, very close approximations can be made.

For small β (see fig. 9(a))

$$\tan \alpha = b/l_r \tag{1}$$

$$\cos \gamma = \frac{l_r + \Delta l}{\sqrt{l_r^2 + b^2}} \tag{2}$$

$$\cos \alpha = \frac{l_r}{\sqrt{l_r^2 + b^2}} \tag{3}$$

$$\sin \alpha = \frac{b}{\sqrt{l_r^2 + b^2}} \tag{4}$$

$$\cos \gamma = \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \tag{5}$$

$$\cos \gamma = \frac{l_r}{\sqrt{l_r^2 + b^2}} \cos \beta + \frac{b}{\sqrt{l_r^2 + b^2}} \sin \beta \tag{6}$$

Then equating equations (2) and (6) yields

$$l_r \cos \beta + b \sin \beta = l_r + \Delta l \tag{7}$$

Substituting

$$\cos \beta = \sqrt{1 - \sin^2 \beta}$$

into equation (7) and solving for $\sin \beta$ give

$$\sin \beta = \frac{2b(l_r + \Delta l) \pm \sqrt{[2b(l_r + \Delta l)]^2 - 4(l_r^2 + b^2)(2l_r \Delta l + \overline{\Delta l^2})}}{2(l_r^2 + b^2)} \tag{8}$$

For small Δl, $\overline{\Delta l^2} << 2l_r\Delta l$

Then

$$\sin \beta = \frac{2bl_r(1 + \zeta) \pm \sqrt{[2bl_r(1 + \zeta)]^2 - 8\zeta l_r^2(l_r^2 + b^2)}}{2(l_r^2 + b^2)} \qquad (9)$$

The condition that $\sin \beta = 0$ when $\zeta = 0$ determines the radical sign as negative:

$$\sin \beta = \frac{2bl_r(1 + \zeta) - \sqrt{[2bl_r(1 + \zeta)]^2 - 8\zeta l_r^2(l_r^2 + b^2)}}{2(l_r^2 + b^2)} \qquad (10)$$

In figures 9(b) and (c)

$$\cos \theta = \frac{a^2 + (r + S)^2 - R^2}{2a(r + S)} \qquad (11)$$

$$b/2 = a \sin \theta \qquad (12)$$

$$b = 2a \sqrt{1 - \cos^2\theta} \qquad (13)$$

$$b = 2a\sqrt{1 - \left[\frac{a^2 + (r + S)^2 - R^2}{2a(r + S)}\right]^2} \qquad (14)$$

Equations (10) and (14) give the solution to the problem.

REFERENCES

1. Hunt, Kenneth C.: Petroleum Requirements of British Gas Turbines. II - Lubricants. SAE Jour., vol. 59, no. 11, Nov. 1951, pp. 20-21.

2. DuBois, Ralph H.: Interesting Applications to Roller Bearings. Presented before Southern New England Section of SAE. (See also discussion, SAE Jour., vol. 60, no. 4, Apr. 1952, pp. 112 and 115.)

3. Boyd, John, and Eklund, P. R.: Some Performance Characteristics of Ball and Roller Bearings for Aviation Gas Turbines. Paper No. 51-A-78, Aviation Div., presented at A.S.M.E. meeting, Atlantic City (N.J.), Nov. 25-30, 1951.

4. Phillips, E. M.: Lubrication - Bearing Problems in Aircraft Gas Turbines. Mech. Eng., vol. 73, no. 12, Dec. 1951, pp. 983-988.

5. Jones, A. B.: The Life of High-Speed Ball Bearings. Trans. A.S.M.E., vol. 74, no. 5, July 1952, pp. 695-699; discussion, pp. 699-703.

6. Dawson, J. G.: Lubricating Problems of the Gas Turbine Engine. Shell Aviation News, no. 133, July 1949, pp. 14-22.

7. Anderson, William J., Macks, E. Fred, and Nemeth, Zolton N.: Comparison of Performance of Experimental and Conventional Cage Designs and Materials for 75-Millimeter-Bore Cylindrical Roller Bearings at High Speeds. NACA Rep. 1177, 1954. (Supersedes NACA TN's 3001 and 3002.)

8. Macks, E. Fred, and Nemeth, Zolton N.: Investigation of 75-Millimeter-Bore Cylindrical Roller Bearings at High Speeds. I - Initial Studies. NACA TN 2128, 1950.

9. Macks, E. Fred, Anderson, W. J., and Nemeth, Zolton N.: Preliminary Comparison of 17- and 75-Millimeter-Bore Cageless Cylindrical Roller Bearings with Conventional Cylindrical Roller Bearings at High Speeds. NACA RM E52L05, 1953.

10. Macks, E. Fred: Preliminary Investigation of Needle Bearings of $1\frac{1}{8}$-Inch Pitch Diameter at Speeds to 17,000 rpm. NACA TN 1920, 1949.

11. Hampp, W. (A. E. Rahm, trans.): Influence of Design Dimensions upon the Operational Performance of Roller Bearings. Translation No. F-TS-604-RE, Air Materiel Command, U.S. Army Air Forces, June 15, 1946.

12. Fogg, A., and Webber, J. S.: The Influence of Some Design Factors on the Characteristics of Ball-Bearings and Roller-Bearings at High Speeds. Proc. Inst. Mech. Eng., vol. 169, no. 36, 1955, pp. 716-731.

13. Macks, E. F., Nemeth, Z. N., and Anderson, W. J.: Operating Characteristics of Cylindrical Roller Bearings at High Speeds. Trans. A.S.M.E., vol. 74, no. 5, July 1952, pp. 705-710; discussion, pp. 711-713.

14. Tarr, Philip R.: Methods for Connection to Revolving Thermocouples. NACA RM E50J23a, 1951.

15. Kreisle, Leonardt F.: Short Journal Bearing Hydrodynamic Performance Under Conditions Approaching Zero Minimum Oil Film Thickness. Paper No. 55-Lub-24, presented at A.S.M.E.-A.S.L.E. Lubrication Conf., Indianapolis (Ind.), Oct. 10-12, 1955.

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TABLE I. - DIMENSIONS AND TEST RESULTS

(a) Roller axial clearance bearings

Bearing	Roller axial clearance, in.	Roller axial clearance ratio	Bearing diametral clearance (unmounted), in.	Cage diametral clearance, in.	Roller clearance in cage pockets (average), in.		Total running time, hr		Remarks
					Axial	Circumferential	Low-speed test	High-speed test	
43	0.0003	0.0005	0.0017	0.0113	0.0117	0.0075	1.3	---	Bearing failed after raising speed to 16,000 rpm. Rollers seized.
44	.0003	.0005	.0017	.0105	.0129	.0090	10.0	2.4	Bearing failed at 22,000 rpm. Rollers seized.
45	.0004	.0006	.0017	.0105	.0116	.0075	7.6	4.8	Satisfactory
48	.0011	.0017	.0018	.0108	.0120	.0090	10.3	4.8	Satisfactory
46	.0014	.0021	.0018	.0110	.0120	.0080	8.0	4.7	Satisfactory
49	.0016	.0024	.0016	.0112	.0145	.0075	8.7	4.3	Satisfactory
50	.0016	.0024	.0017	.0104	.0145	.0100	9.7	6.3	Satisfactory
52	.0021	.0032	.0017	.0105	.0141	.0090	10.6	6.7	Satisfactory
53	.0023	.0035	.0016	.0105	.0145	.0085	10.5	3.2	Satisfactory
54	.0023	.0035	.0016	.0103	.0150	.0085	10.8	5.9	Satisfactory
57	.0029	.0044	.0018	.0103	.0145	.0095	11.6	6.1	Satisfactory
56	.0030	.0046	.0018	.0104	.0140	.0090	9.1	6.0	Satisfactory
59	.0035	.0053	.0020	.0106	.0145	.0090	10.3	4.2	Satisfactory
60	.0040	.0061	.0018	.0109	.0145	.0095	9.9	5.9	Satisfactory

TABLE I. - Continued. DIMENSIONS AND TEST RESULTS

(b) Cage diametral clearance bearings
(nominal cage land width, 0.130 in.)

Bearing	Cage diametral clearance, in.	Cage diametral clearance ratio	Bearing diametral clearance (unmounted), in.	Roller clearance in cage pockets (nominal), in.		Roller axial clearance, in.	Total running time, hr	Remarks
				Axial	Circumferential			
61	0.004	0.0011	0.0015	0.011	0.011	0.0024	21.7	Brass pickup on inner race; cage pocket wear, light
65	.004	.0011	.0020	.011	.011	.0020	21.7	Slight brass pickup on inner race; cage pocket wear, light
67	.008	.0022	.0020	.011	.011	.0021	22.2	Cage pocket wear, light
62	.009	.0024	.0018	.011	.011	.0032	22.1	Brass pickup on inner race; cage pocket wear, light to medium
68	.012	.0032	.0018	.011	.011	.0018	22.1	Brass pickup on inner race; cage pocket wear, medium to heavy
64	.013	.0035	.0018	.011	.011	.0020	22.5	Brass pickup on inner race; cage pocket wear, light to heavy
73	.013	.0035	.0017	.011	.011	.0029	20.0	Brass pickup on inner race; cage pocket wear, heavy
66	.016	.0043	.0016	.011	.011	.0031	20.0	Brass pickup on inner race; cage pocket wear, medium to heavy
63	.021	.0057	.0015	.011	.011	.0020	22.1	Brass pickup on inner race; cage pocket wear, medium to heavy
69	.022	.0059	.0019	.011	.011	.0018	21.4	Slight brass pickup on inner race; cage pocket wear, medium to heavy

TABLE I. - Concluded.   DIMENSIONS AND TEST RESULTS

(c) Cage land width bearings

Bearing	Cage land width, in.	Cage land width-diameter ratio	Cage diametral clearance, in.	Bearing diametral clearance (unmounted), in.	Roller clearance in cage pockets (nominal), in.		Roller axial clearance, in.	Total running time, hr	Remarks
					Axial	Circumferential			
62	0.130	0.035	0.009	0.0018	0.011	0.011	0.0032	22.1	Brass pickup on inner race; cage pocket wear, light to medium
67	.130	.035	.008	.0020	.011	.011	.0021	22.2	Cage pocket wear, light
72	.151	.041	.009	.0017	.011	.011	.0023	21.8	Cage pocket wear, medium to heavy
78	.151	.041	.009	.0020	.011	.011	.0015	----	Slight brass pickup on inner race; cage pocket wear, light to medium
74	.171	.046	.009	.0018	.011	.011	.0024	22.6	Cage pocket wear, medium to heavy
75	.171	.046	.009	.0018	.011	.011	.0026	22.0	Cage pocket wear, medium to heavy
71	.192	.052	.009	.0017	.011	.011	.0027	21.9	Cage pocket wear, medium to very heavy
77	.192	.052	.010	.0020	.011	.011	.0022	21.2	Trace of brass pickup on inner race; cage pocket wear, medium to heavy

TABLE II. - RUNNING TIME FOR ROLLER

AXIAL CLEARANCE BEARINGS

[Radial load, 368 lb]

(a) Low-speed tests

Bearing	Running time, hr, at DN value of -			
	0.3×10^6	0.735×10^6	0.995×10^6	1.2×10^6
[a]43	0.75	0.25	0.25	0.03
44	3.91	2.16	1.83	2.03
45	2.66	1.75	1.58	1.60
48	5.25	1.66	1.66	1.75
46	2.83	1.58	1.58	2.0
49	2.35	2.5	1.75	2.05
50	3.33	2.16	2.08	2.16
52	4.01	2.5	1.66	2.48
53	3.91	2.2	2.5	1.9
54	4.83	1.75	2.16	2.15
57	3.67	2.67	2.64	2.63
56	3.52	1.83	1.92	1.8
59	3.25	2.1	2.26	2.64
60	3.25	2.31	2.11	2.25

[a]Failed at DN value of 1.2×10^6.

TABLE II. - Concluded. RUNNING TIME FOR ROLLER AXIAL CLEARANCE BEARINGS

[Radial load, 368 lb]

(b) High-speed tests

Bearing	Running time, hr, at DN value of -												
	1.2×10^6	1.275×10^6	1.35×10^6	1.425×10^6	1.5×10^6	1.575×10^6	1.65×10^6	1.725×10^6	1.8×10^6	1.875×10^6	1.95×10^6	2.025×10^6	2.1×10^6
[a]44	0.50	0.17	0.33	0.17	0.33	0.17	0.33	0.17	0.17	----	----	----	----
45	1.5	.17	.31	.13	.81	.3	.58	.37	.35	0.12	0.15	----	----
48	1.67	.14	.27	.15	.59	.25	.52	.27	.23	.12	.59	----	----
46	1.42	.34	.3	.25	.64	.25	.61	.29	.29	.12	.25	----	----
49	1.33	.18	.35	.14	.58	.25	.35	.12	.43	.25	.33	----	----
50	2.65	.16	.35	.16	.67	.33	.71	.12	.6	.12	.43	----	----
52	2.67	.2	.8	.16	.67	.31	.72	.25	.31	.31	.31	----	----
53	.75	.08	.27	.25	.48	.65	.50	.08	.08	----	----	----	----
54	2.23	.23	.35	.16	.68	.33	.59	.16	.44	.32	.35	----	----
57	1.68	.4	.15	.2	.67	.17	.46	.13	.75	.12	.50	0.27	0.60
56	1.42	.26	.14	.15	.65	.29	.54	.26	.52	.28	.78	.29	.38
59	1.83	.13	.28	.11	.56	.13	.5	.13	.23	.11	.22	----	----
60	2.97	.13	.27	.22	.64	.3	.35	.15	.42	.13	.29	----	----

[a]Failed at DN value of 1.65×10^6 during second run.

TABLE III. - ROLLER AXIAL CLEARANCE BEARINGS

(a) Cage-wear data obtained before cage-slip tests

Bearing	Roller axial clearance ratio	Roller plus inner-race wear, mg	
		Low-speed tests[a]	High-speed tests[b]
45	0.0006	3	378
48	.0017	167	232
46	.0021	231	324
49	.0024	331	340
50	.0024	351	46
52	.0032	311	355
53	.0035	331	518
54	.0035	282	292
57	.0044	366	146
56	.0046	222	284
59	.0053	272	138
60	.0061	302	114

[a]DN values from 0.3×10^6 to 1.2×10^6.
[b]DN values from 1.2×10^6 to 2.1×10^6.

(b) Cage-slip data (oil flow, 2.75 lb/min)

Bearing	Roller axial clearance ratio	Cage slip, percent, at DN value of -						Remarks
		1.2×10^6 (16,000 rpm)			1.5×10^6 (20,000 rpm)			
		Load, lb						
		7	113	368	7	113	368	
46	0.0021	60	52	13 - 16	45	44	21 - 23	Satisfactory
50	.0024	55	47	16 - 18	63	53	22	Satisfactory
53	.0035	46	15	8	39	29	7	Satisfactory
54	.0035	40	33	4	40	40	11	Satisfactory
59	.0053	65	50	6 - 10	--	52	14 - 22	Satisfactory
60	.0061	50	50	20	55	58	18	Satisfactory

TABLE IV. - CAGE DIAMETRAL

CLEARANCE BEARINGS

(a) Cage-wear data obtained
before cage-slip tests

Bearing	Cage diametral clearance ratio	Cage wear, mg[a]
61	0.0011	60
65	.0011	86
67	.0022	55
62	.0024	111
68	.0032	285
64	.0035	170
73	.0035	356
66	.0043	162
63	.0057	322
69	.0059	119

[a]DN values from 0.3×10^6 to 1.2×10^6.

(b) Cage-slip data (oil flow, 2.75 lb/min)

Bearing	Cage diametral clearance ratio	Cage slip, percent, at DN value of						Remarks
		1.2×10^6 (16,000 rpm)			1.5×10^6 (20,000 rpm)			
		Load, lb						
		7	113	368	7	113	368	
61	0.0011	12	14	8	10	10	9	Satisfactory
65	.0011	10	10	9	9	10	9	Satisfactory
67	.0022	62	42	6	4	5	8	Operation very rough at DN 1.5×10^6 (20,000 rpm) and 7- and 113-lb loads
62	.0024	12 - 68	15	10	8	9	10	Satisfactory
68	.0032	11	14	10	9	11	10	Ran all right at DN of 1.8×10^6 (24,000 rpm)
64	.0035	42	14	9	9	7	8	Ran all right at DN of 1.8×10^6 (24,000 rpm)
63	.0057	8 - 59	9	8	--	--	9	Incipient failure at 113-lb load and DN of 1.5×10^6 (20,000 rpm)
69	.0059	47	35	6	9	--	--	Incipient failure at DN of 1.5×10^6 (20,000 rpm)

TABLE V. - CAGE LAND WIDTH BEARINGS

(a) Cage-wear data obtained
before cage-slip tests

Bearing	Cage land width-diameter ratio	Cage wear, mg[a]
62	0.035	111
67	.035	55
72	.041	63
78	.041	132
74	.046	223
75	.046	223
71	.052	67
77	.052	274

[a]DN values from 0.3×10^6 to 1.2×10^6.

(b) Cage-slip data (oil flow, 2.75 lb/min)

Bearing	Cage land width-diameter ratio	Cage slip, percent, at DN value of -						Remarks
		1.2×10^6 (16,000 rpm)			1.5×10^6 (20,000 rpm)			
		Load, lb						
		7	113	368	7	113	368	
62	0.035	12 - 68	15	10	8	9	10	Satisfactory
67	.035	62	42	6	4	5	8	Operation very rough at DN of 1.5×10^6 (20,000 rpm) and 113-lb load
72	.041	0	0	-4	0	1	0	Satisfactory
78	.041	4	2	2	5	4	5	Satisfactory
74	.046	6	5	5	3	4	5	Satisfactory
75	.046	-6	-9	-9	-	-	3	Failed at DN of 1.5×10^6 (20,000 rpm) and 113-lb load
71	.052	6	11	7 - 9	-	-	--	Incipient failure at DN of 1.5×10^6 (20,000 rpm) and 368-lb load
77	.052	11	7	8	-	-	--	Failed at DN of 1.5×10^6 (20,000 rpm) and 368-lb load
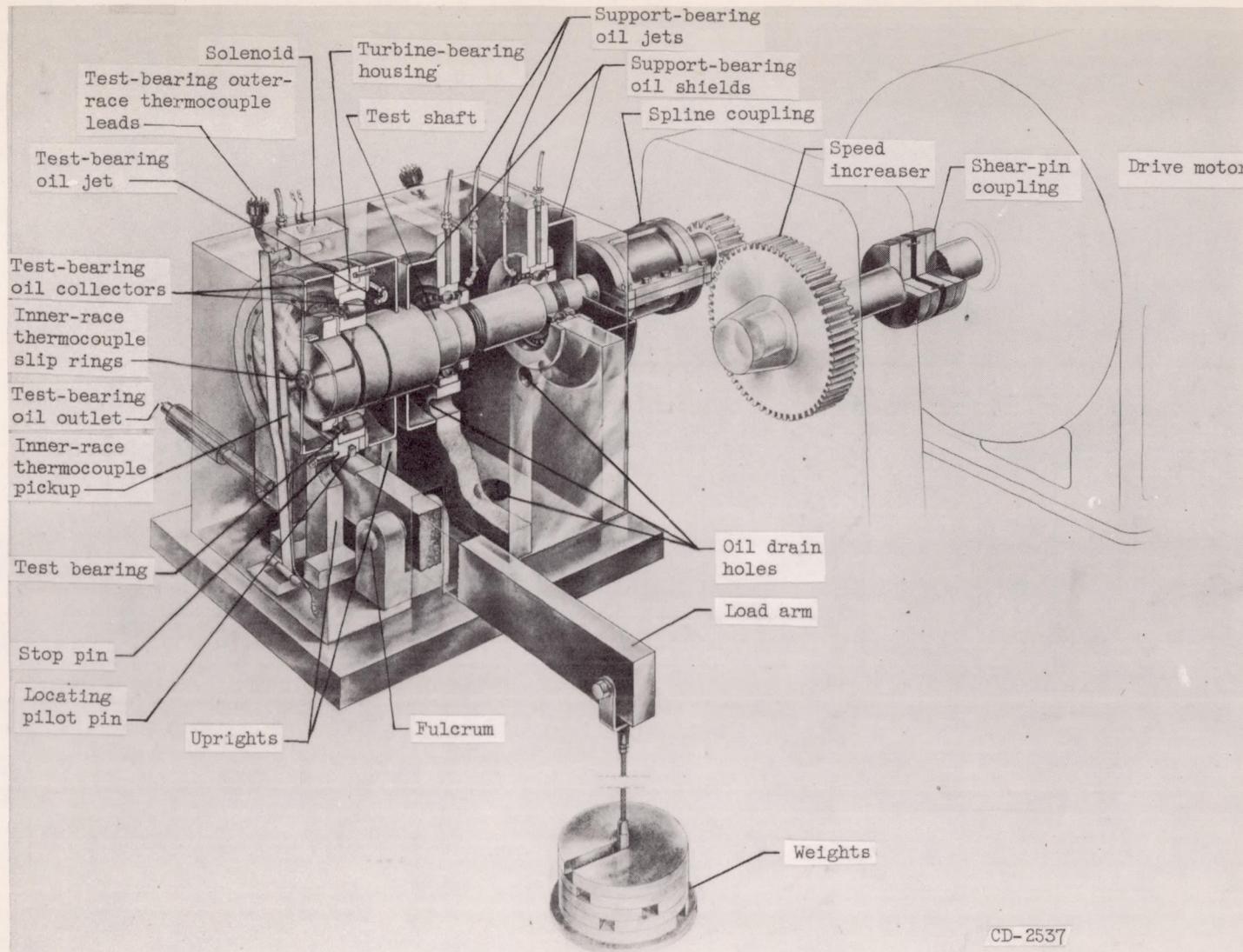
Figure 1. - Cutaway view of radial-load rig.
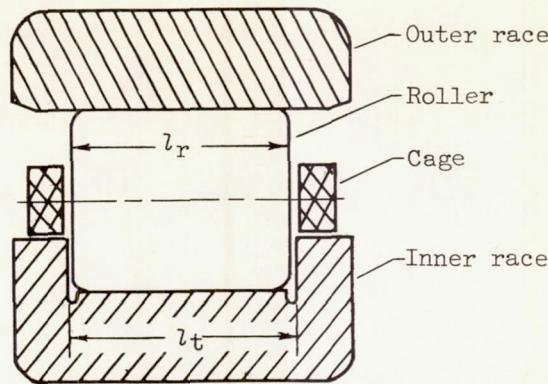
(a) Roller axial clearance bearings. Roller axial clearance ratio = $\dfrac{l_t - l_r}{l_r}$.
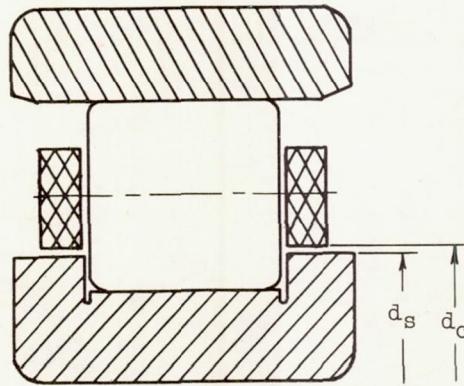
(b) Cage diametral clearance bearings. Cage diametral clearance ratio = $\dfrac{d_c - d_s}{d_c}$.
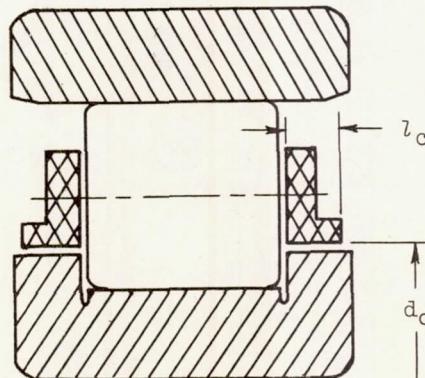
(c) Cage land width bearings. Cage land width-diameter ratio = $\dfrac{l_c}{d_c}$.
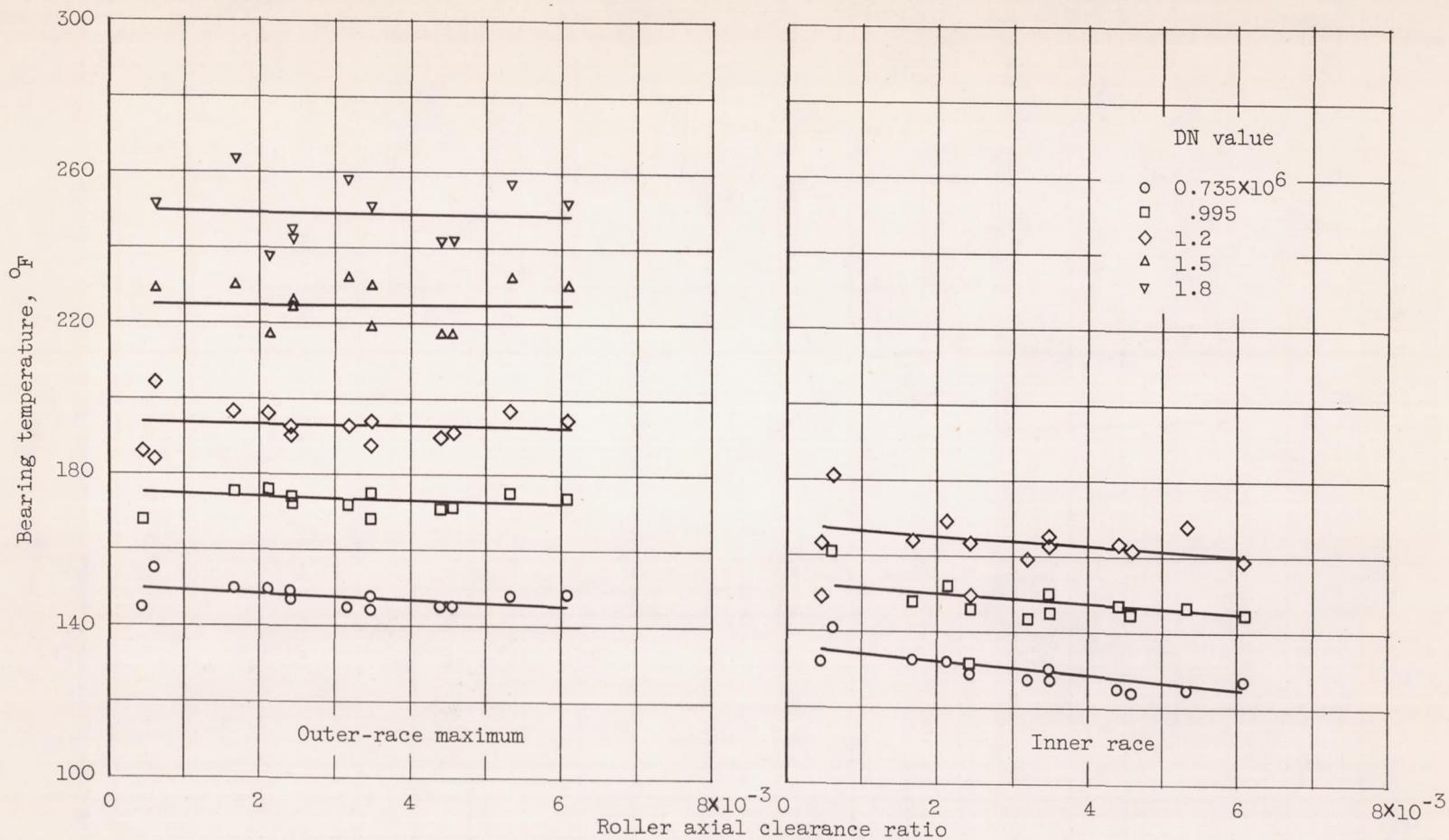
Figure 2. - Schematic diagrams of test bearings.
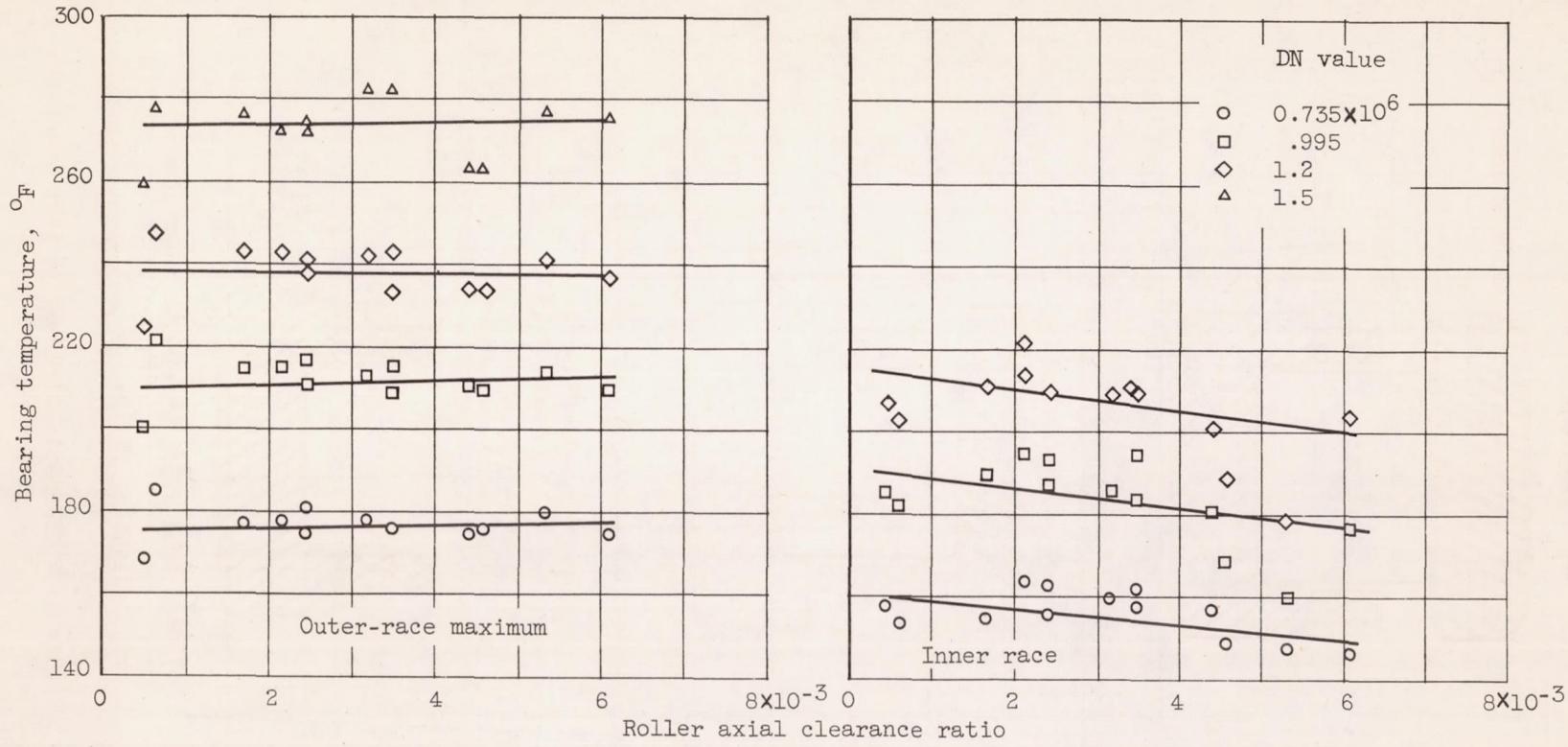
(a) Oil flow, 2.0 pounds per minute.

Figure 3. - Effect of roller axial clearance ratio on bearing outer-race-maximum and inner-race temperatures. Load, 368 pounds; oil inlet temperature, 100° F.
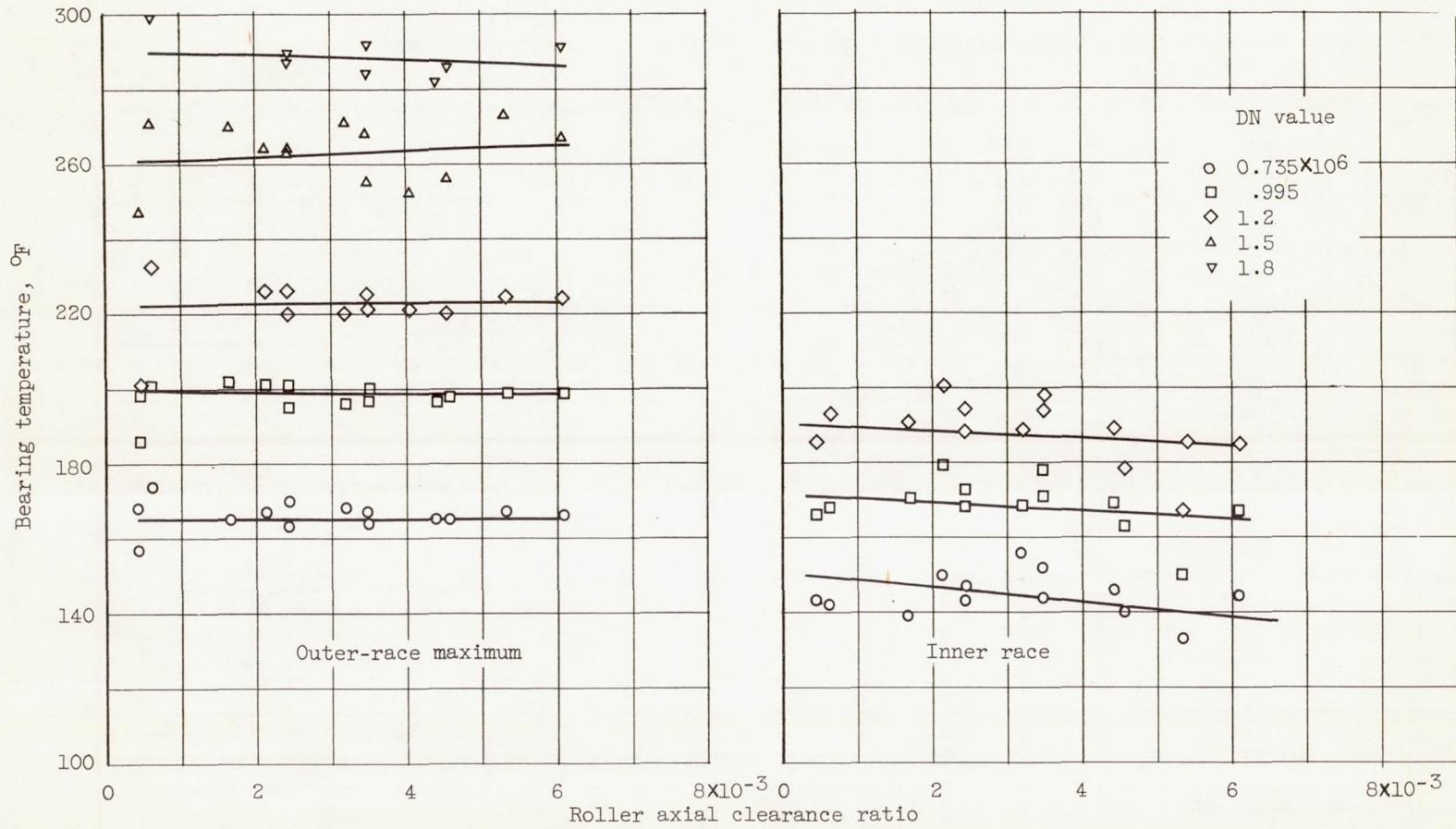
(b) Oil flow, 2.75 pounds per minute.

Figure 3. - Continued. Effect of roller axial clearance ratio on bearing outer-race-maximum and inner-race temperatures. Load, 368 pounds; oil inlet temperature, 100° F.
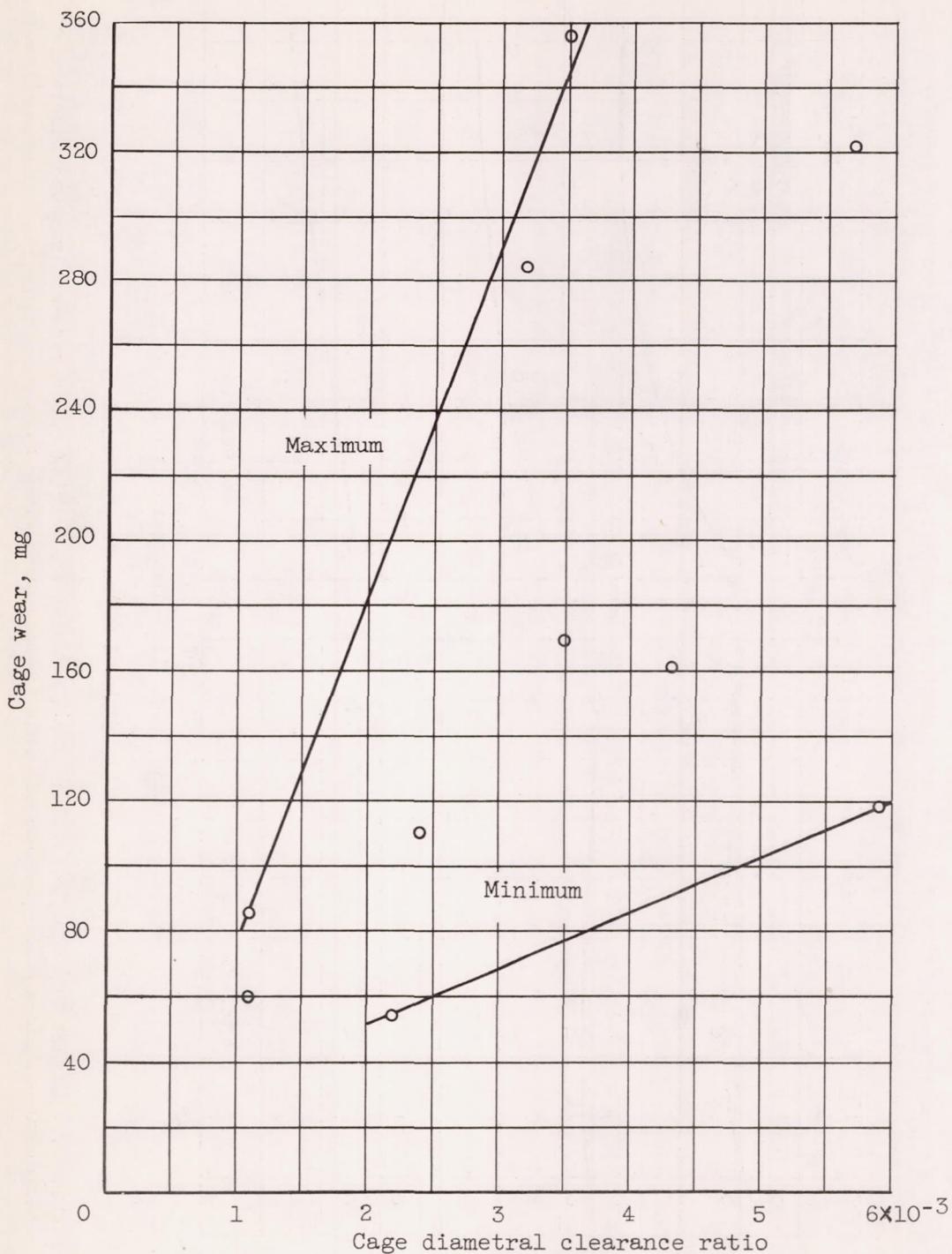
(c) Oil flow, 5.5 pounds per minute.
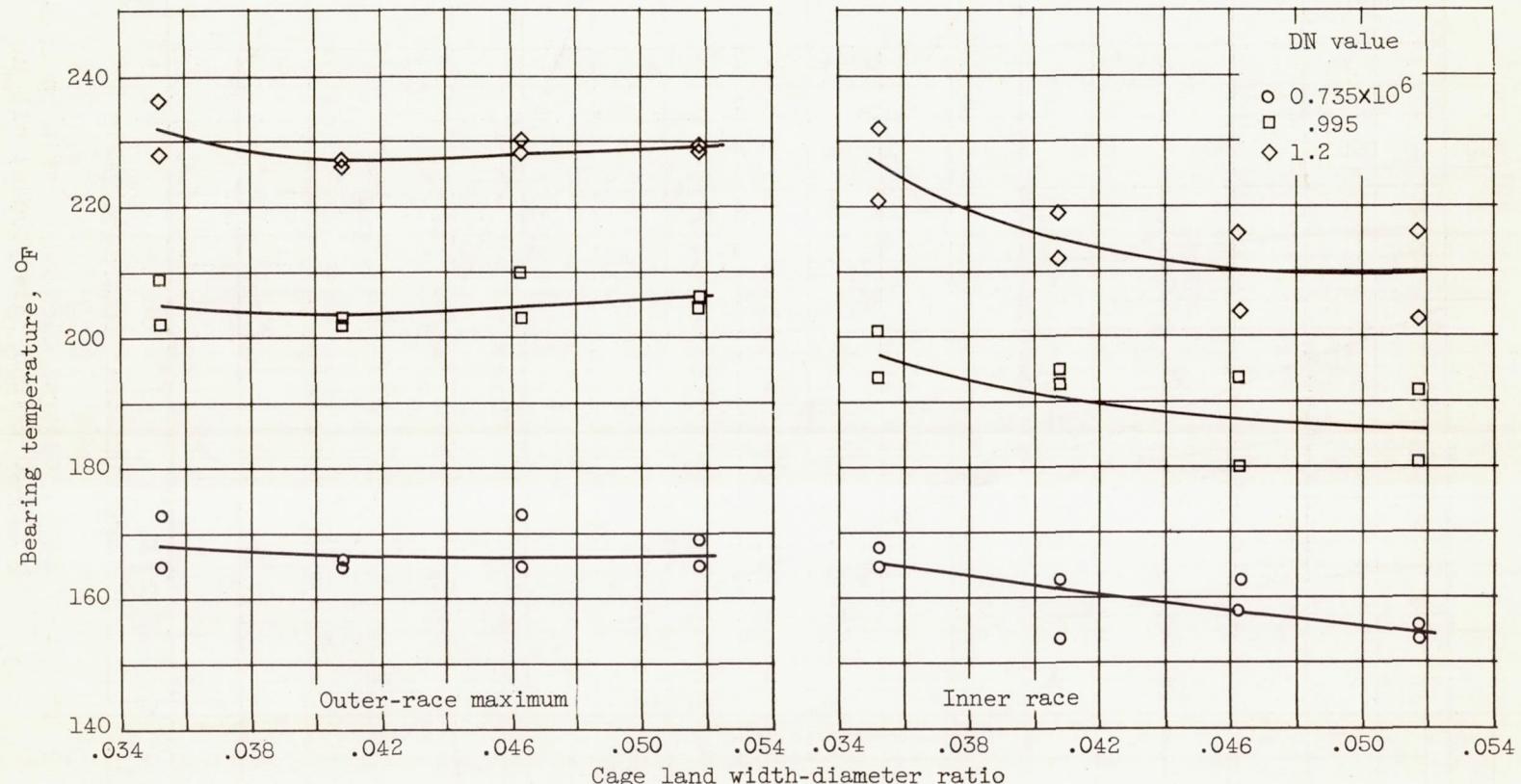
Figure 3. - Concluded. Effect of roller axial clearance ratio on bearing outer-race-maximum and inner-race temperatures. Load, 368 pounds; oil inlet temperature, 100° F.
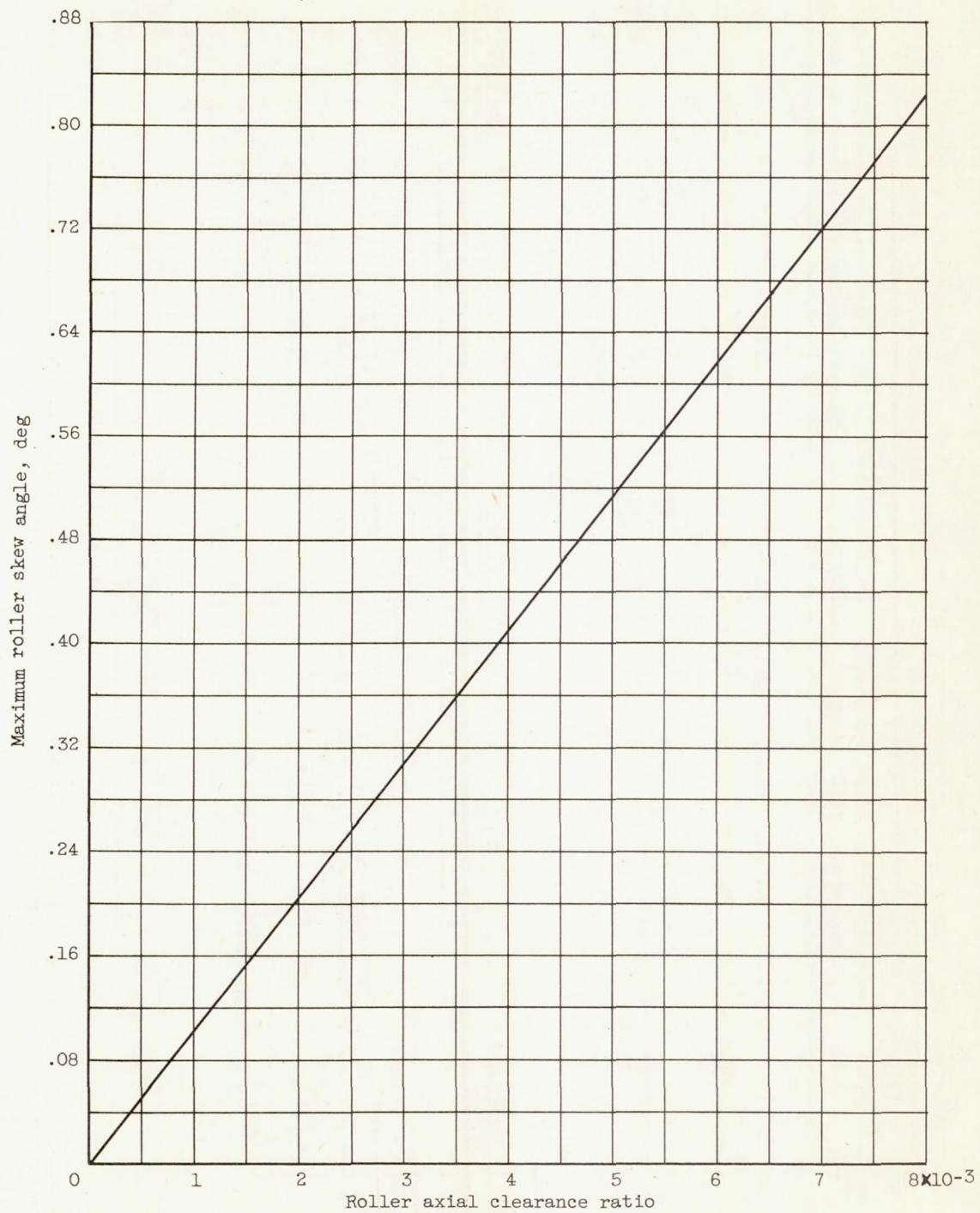
Figure 4. - Maximum roller skew angle as a function of roller axial clearance ratio for roller axial clearance bearings (based on derivation of appendix B).
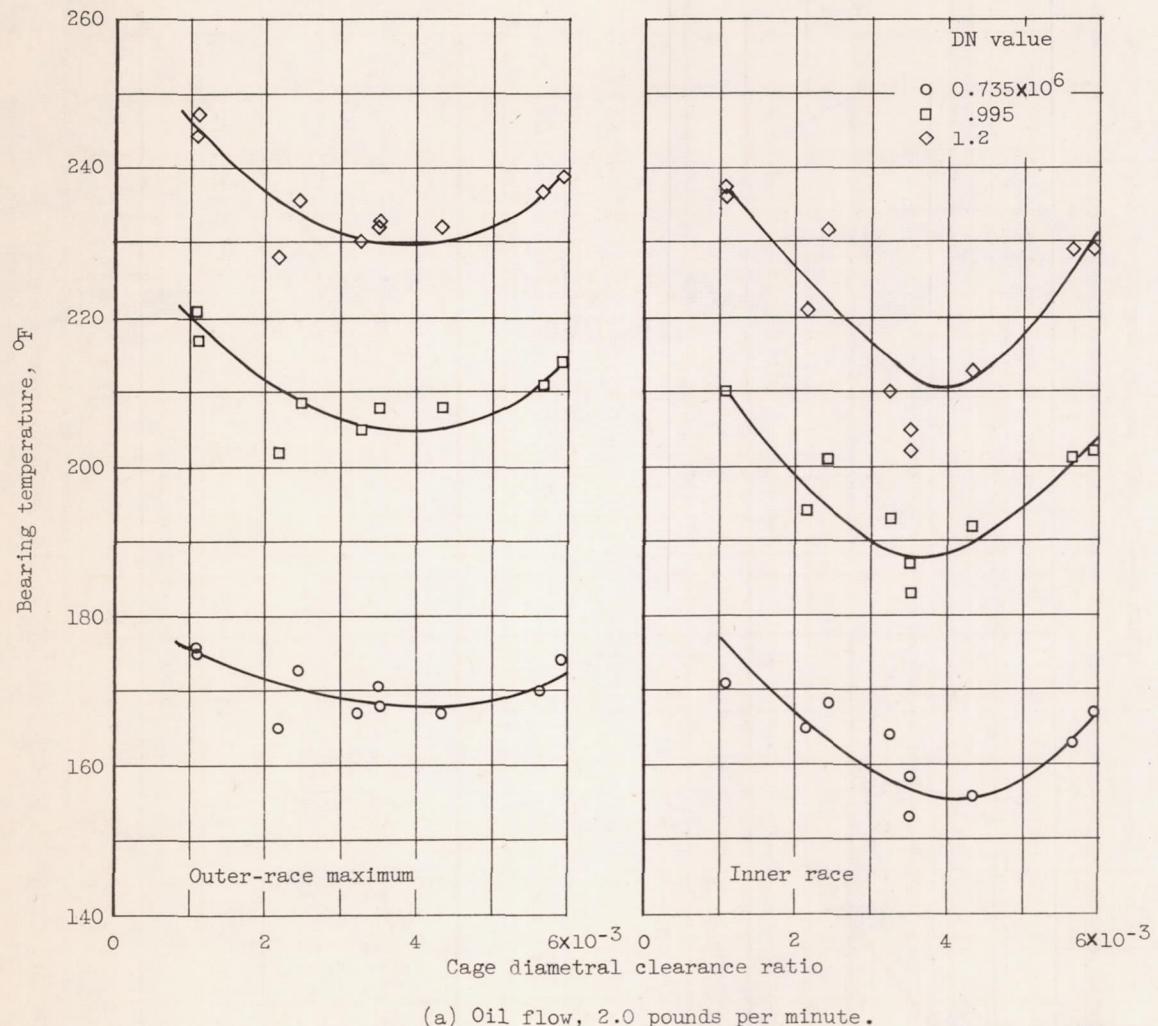
(a) Oil flow, 2.0 pounds per minute.

Figure 5. - Effect of cage diametral clearance ratio on bearing outer-race-maximum and inner-race temperatures. Load, 368 pounds; oil inlet temperature, 100° F.
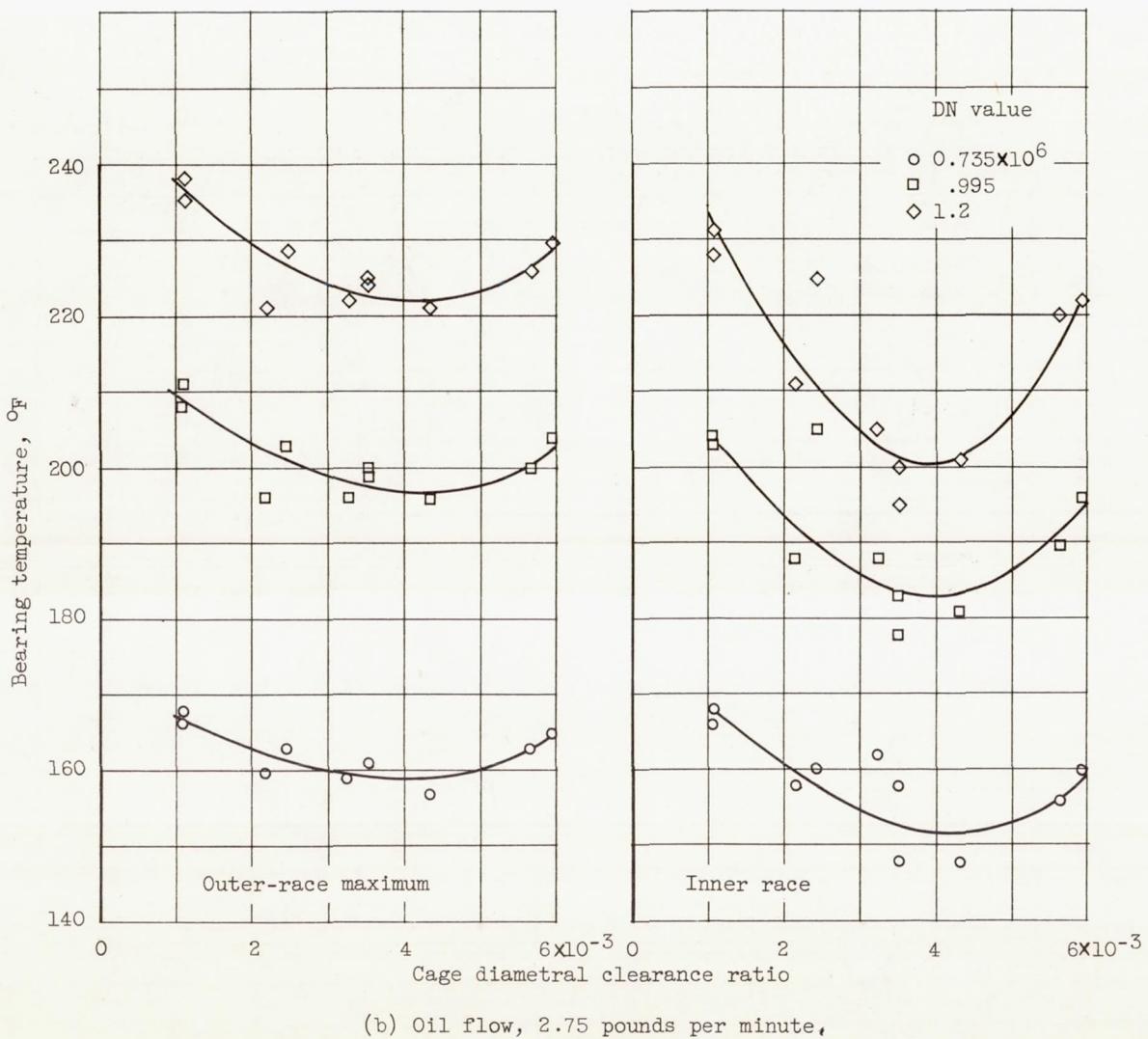
(b) Oil flow, 2.75 pounds per minute,

Figure 5. - Concluded. Effect of cage diametral clearance ratio on bearing outer-race-maximum and inner-race temperatures. Load, 368 pounds; oil inlet temperature, 100° F

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Figure 6. - Effect of cage diametral clearance ratio on cage wear. DN values, 0.3×10^6 to 1.2×10^6; load, 368 pounds; oil inlet temperature, 100° F; running time, 20 hours.

(a) Oil flow, 2.0 pounds per minute.

Figure 7. - Effect of cage land width-diameter ratio on bearing outer-race-maximum and inner-race temperatures. Load, 368 pounds; oil inlet temperature, 100° F.
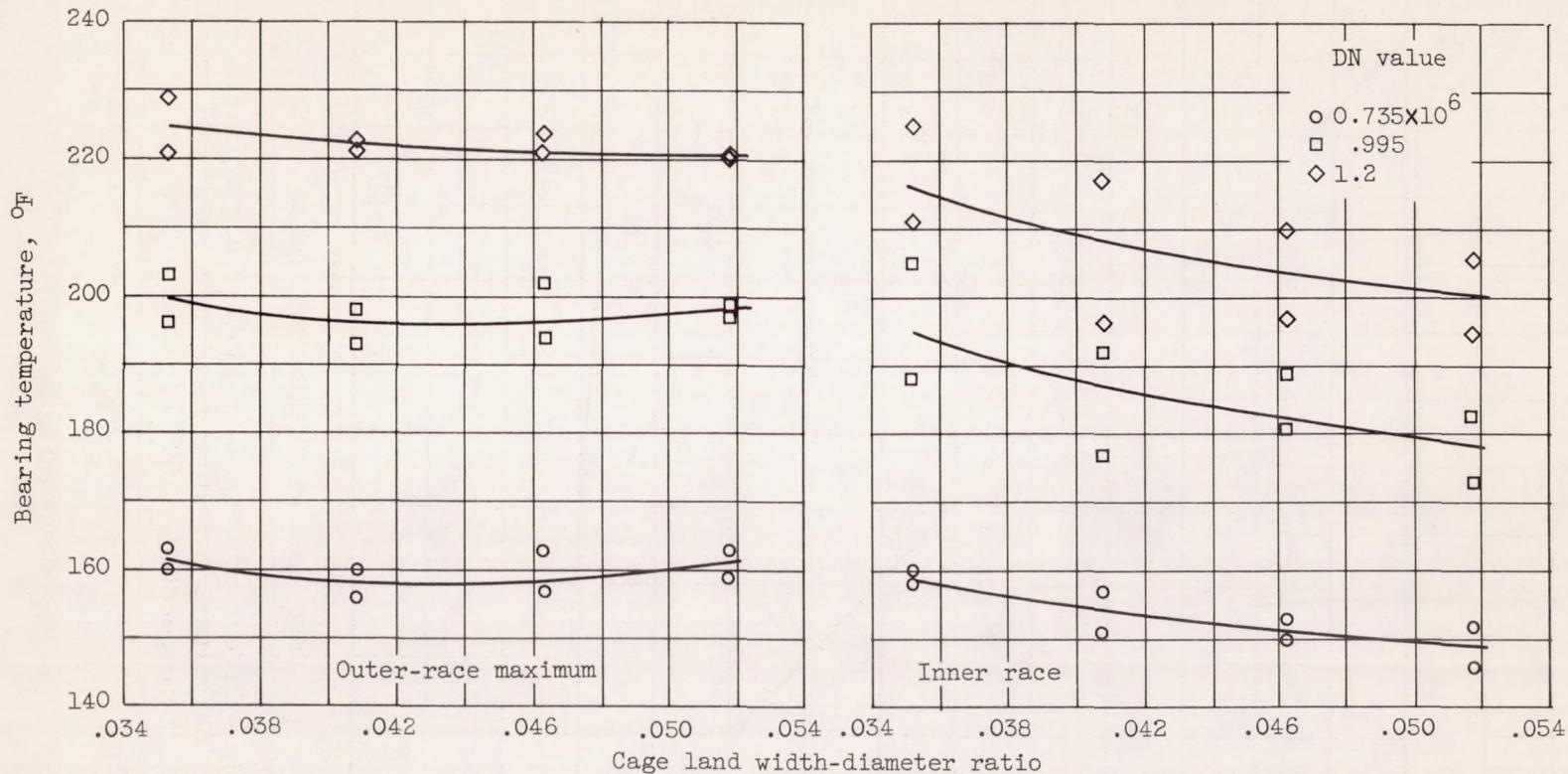
(b) Oil flow, 2.75 pounds per minute.

Figure 7. - Concluded. Effect of cage land width-diameter ratio on bearing outer-race-maximum and inner-race temperatures. Load, 368 pounds; oil inlet temperature, 100° F.
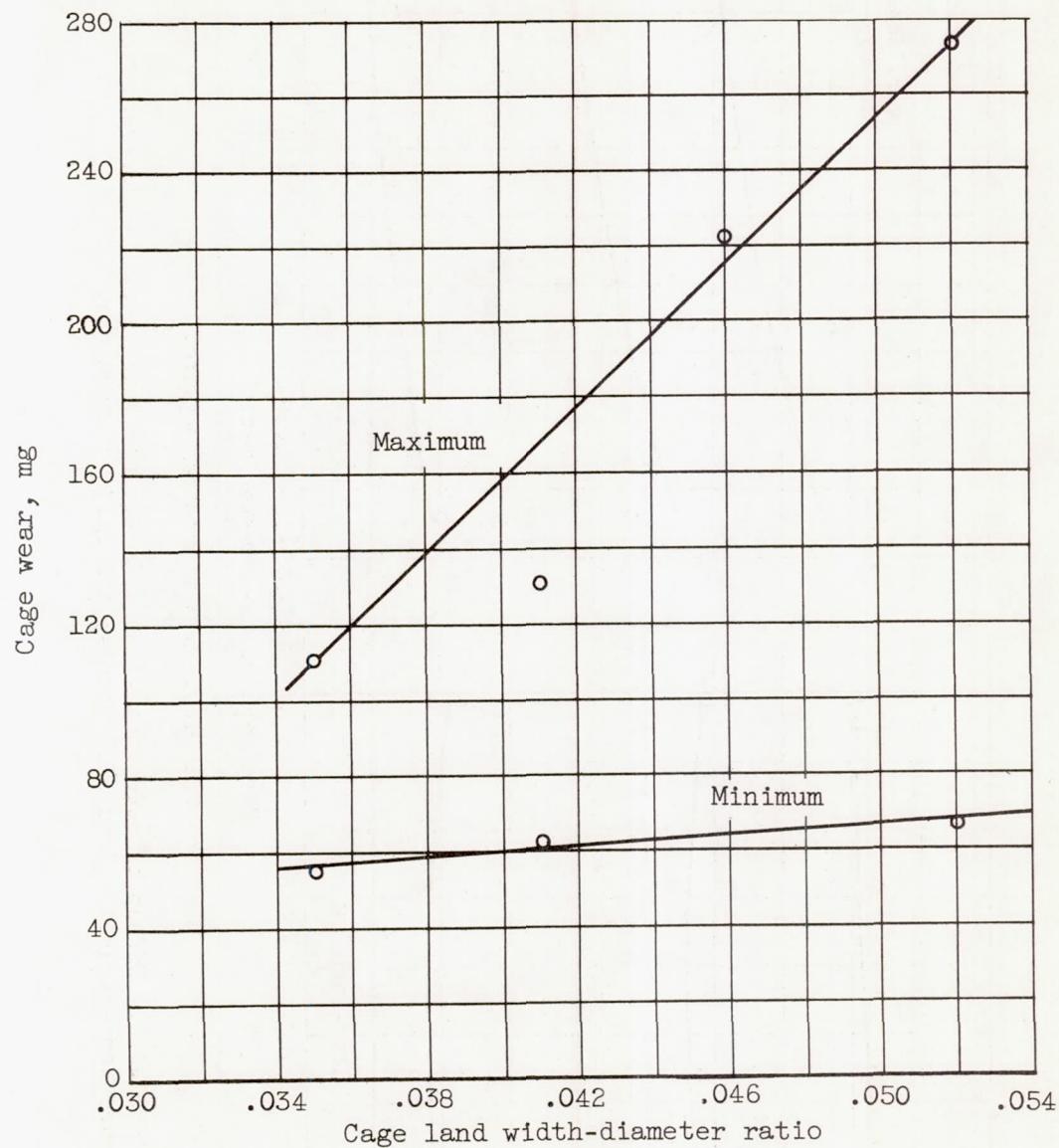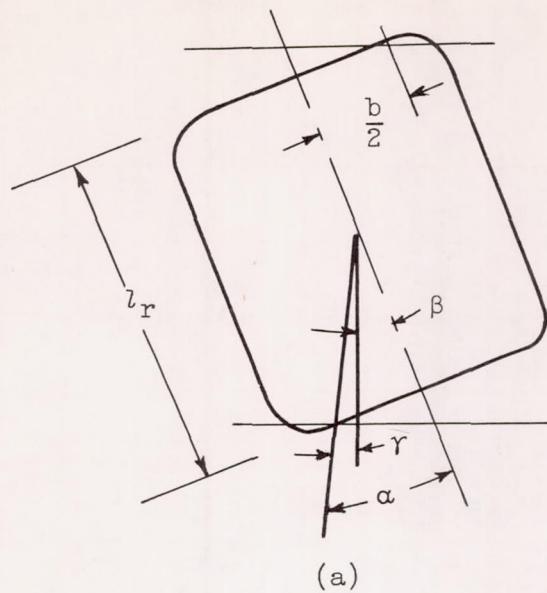
Figure 8. - Effect of cage land width-diameter ratio on cage
    wear. DN values, 0.3×10^6 to 1.2×10^6; load, 368 pounds;
    oil inlet temperature, 100° F.
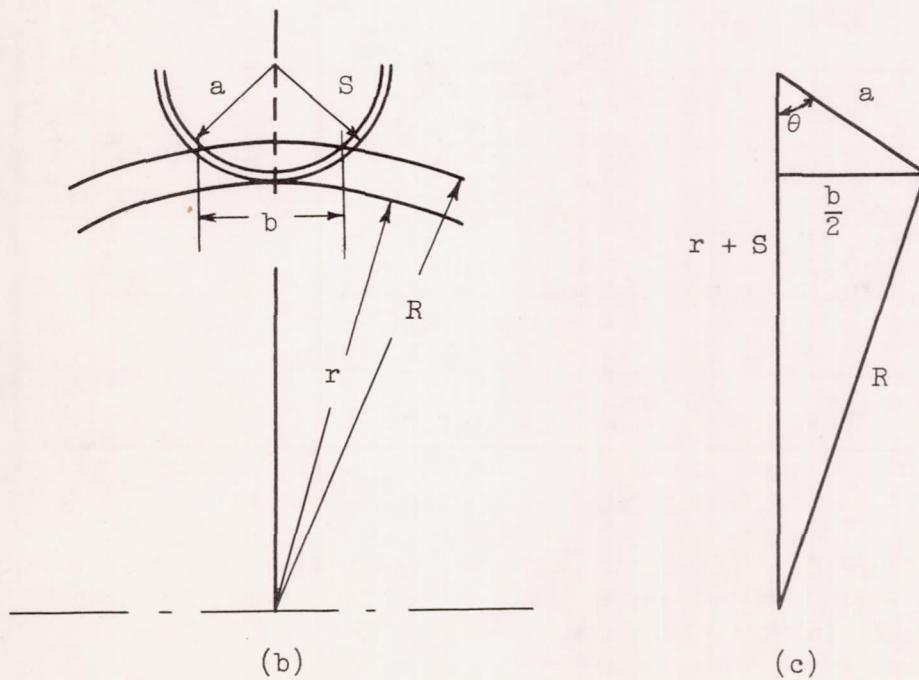
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(a)

(b)   (c)

Figure 9. - Relations used in derivation of roller
       skew angle.